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STANDARD EQUATIONS OF MOTION FOR SUBMARINE SIMULATION

by

Morton Gertler
and
Grant R. Hagen

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NOTATION

Symbol	Dimensionless Form	Definition
B	$B' = \frac{B}{\frac{1}{2}\rho \ell^2 U^2}$	Buoyancy force, positive upward
CB		Center of buoyancy of submarine
CG		Center of mass of submarine
I_x	$I_x' = \frac{I_x}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about x axis
I_y	$I_y' = \frac{I_y}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about y axis
I_z	$I_z' = \frac{I_z}{\frac{1}{2}\rho \ell^5}$	Moment of inertia of submarine about z axis
I_{xy}	$I_{xy}' = \frac{I_{xy}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about xy axis
I_{yz}	$I_{yz}' = \frac{I_{yz}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about yz axes
I_{zx}	$I_{zx}' = \frac{I_{zx}}{\frac{1}{2}\rho \ell^5}$	Product of inertia about zx axes
K	$K' = \frac{K}{\frac{1}{2}\rho \ell^3 U^2}$	Hydrodynamic moment component about x axis (rolling moment)
K_*	$K_*' = \frac{K_*}{\frac{1}{2}\rho \ell^3 U^2}$	Rolling moment when body angle (α, β) and control surface angles are zero
$K_{*\eta}$	$K_{*\eta}' = \frac{K_{*\eta}}{\frac{1}{2}\rho \ell^3 U^2}$	Coefficient used in representing K_* as a function of $(\eta-1)$
K_p	$K_p' = \frac{K_p}{\frac{1}{2}\rho \ell^4 U}$	First order coefficient used in representing K as a function of p
$K_{\dot{p}}$	$K_{\dot{p}}' = \frac{K_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of $\dot{p}$
$K_{p\|p\|}$	$K_{p\|p\|}' = \frac{K_{p\|p\|}}{\frac{1}{2}\rho \ell^5}$	Second order coefficient used in representing K as a function of p
K_{pq}	$K_{pq}' = \frac{K_{pq}}{\frac{1}{2}\rho \ell^5}$	Coefficient used in representing K as a function of the product pq

K_{qr}	$K_{qr}' = \frac{K_{qr}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of the product qr
K_r	$K_r' = \frac{K_r}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing K as a function of r
$K_{\dot{r}}$	$K_{\dot{r}}' = \frac{K_{\dot{r}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of $\dot{r}$
K_v	$K_v' = \frac{K_v}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing K as a function of v
$K_{\dot{v}}$	$K_{\dot{v}}' = \frac{K_{\dot{v}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of $\dot{v}$
$K_{v\|v\|}$	$K_{v\|v\|}' = \frac{K_{v\|v\|}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing K as a function of v
K_{vq}	$K_{vq}' = \frac{K_{vq}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of the product vq
K_{vw}	$K_{vw}' = \frac{K_{vw}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K as a function of the product vw
K_{wp}	$K_{wp}' = \frac{K_{wp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of the product wp
K_{wr}	$K_{wr}' = \frac{K_{wr}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of the product wr
$K_{\delta r}$	$K_{\delta r}' = \frac{K_{\delta r}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing K as a function of δ_r
ℓ	$\ell' = 1$	Overall length of submarine
m	$m' = \frac{m}{\frac{1}{2}\rho\ell^3}$	Mass of submarine, including water in free-flooding spaces
M	$M' = \frac{M}{\frac{1}{2}\rho\ell^3 U^2}$	Hydrodynamic moment component about y axis (pitching moment)
M_*	$M_*' = \frac{M_*}{\frac{1}{2}\rho\ell^3 U^2}$	Pitching moment when body angles (α, β) and control surface angles are zero
M_{pp}	$M_{pp}' = \frac{M_{pp}}{\frac{1}{2}\rho\ell^5}$	Second order coefficient used in representing M as a function of p. First order coefficient is zero.
M_q	$M_q' = \frac{M_q}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing M as a function of q
$M_{q\eta}$	$M_{q\eta}' = \frac{M_{q\eta}}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing M_q as a function of (η-1)
$M_{\dot{q}}$	$M_{\dot{q}}' = \frac{M_{\dot{q}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing M as a function of $\dot{q}$

$M_{q\|q\|}$	$M_{q\|q\|}' = \dfrac{M_{q\|q\|}}{\frac{1}{2}\rho\ell^5}$	Second order coefficient used in representing M as a function of q
$M_{\|q\|\delta s}$	$M_{\|q\|\delta s}' = \dfrac{M_{\|q\|\delta s}}{\frac{1}{2}\rho\ell^4 U}$	Coefficient used in representing $M_{\delta s}$ as a function q
M_{rp}	$M_{rp}' = \dfrac{M_{rp}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing M as a function of the product rp
M_{rr}	$M_{rr}' = \dfrac{M_{rr}}{\frac{1}{2}\rho\ell^5}$	Second order coefficient used in representing M as a function of r. First order coefficient is zero
M_{vp}	$M_{vp}' = \dfrac{M_{vp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M as a function of the product vp
M_{vr}	$M_{vr}' = \dfrac{M_{vr}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M as a function of the product vr
M_{vv}	$M_{vv}' = \dfrac{M_{vv}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing M as a function of v
M_w	$M_w' = \dfrac{M_w}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing M as a function of w
$M_{w\eta}$	$M_{w\eta}' = \dfrac{M_{w\eta}}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing M_w as a function of $(\eta-1)$
$M_{\dot{w}}$	$M_{\dot{w}}' = \dfrac{M_{\dot{w}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M as a function of $\dot{w}$
$M_{\|w\|}$	$M_{\|w\|}' = \dfrac{M_{\|w\|}}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing M as a function of w; equal to zero for symmetrical function
$M_{\|w\|q}$	$M_{\|w\|q}' = \dfrac{M_{\|w\|q}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M_q as a function of w
$M_{w\|w\|}$	$M_{w\|w\|}' = \dfrac{M_{w\|w\|}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing M as a function of w
$M_{w\|w\|\eta}$	$M_{w\|w\|\eta}' = \dfrac{M_{w\|w\|\eta}}{\frac{1}{2}\rho\ell^3}$	First order coefficient used in representing $M_{w\|w\|}$ as a function of $(\eta-1)$
M_{ww}	$M_{ww}' = \dfrac{M_{ww}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing M as a function of w; equal to zero for symmetrical function
$M_{\delta b}$	$M_{\delta b}' = \dfrac{M_{\delta b}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing M as a function of δ_b
$M_{\delta s}$	$M_{\delta s}' = \dfrac{M_{\delta s}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing M as a function of δ_s
$M_{\delta s\eta}$	$M_{\delta s\eta}' = \dfrac{M_{\delta s\eta}}{\frac{1}{2}\rho\ell^3 U^2}$	First order coefficient used in representing $M_{\delta s}$ as a function of $(\eta-1)$

N	$N' = \frac{N}{\frac{1}{2}\rho\ell^3 U^2}$	Hydrodynamic moment component about z axis (yawing moment)
N_*	$N_*' = \frac{N_*}{\frac{1}{2}\rho\ell^3 U^2}$	Yawing moment when body angles (α, β) and control surface angles are zero
N_p	$N_p' = \frac{N_p}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing N as a function of p
$N_{\dot{p}}$	$N_{\dot{p}}' = \frac{N_{\dot{p}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of $\dot{p}$
N_{pq}	$N_{pq}' = \frac{N_{pq}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of the product pq
N_{qr}	$N_{qr}' = \frac{N_{qr}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of the product qr
N_r	$N_r' = \frac{N_r}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing N as a function of r
$N_{r\eta}$	$N_{r\eta}' = \frac{N_{r\eta}}{\frac{1}{2}\rho\ell^4 U}$	First order coefficient used in representing N_r as a function of $(\eta - 1)$
$N_{\dot{r}}$	$N_{\dot{r}}' = \frac{N_{\dot{r}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of $\dot{r}$
$N_{r\|r\|}$	$N_{r\|r\|}' = \frac{N_{r\|r\|}}{\frac{1}{2}\rho\ell^5}$	Second order coefficient used in representing N as a function of r
$N_{\|r\|\delta r}$	$N_{\|r\|\delta r}' = \frac{N_{\|r\|\delta r}}{\frac{1}{2}\rho\ell^4 U}$	Coefficient used in representing $N_{\delta r}$ as a function of r
N_v	$N_v' = \frac{N_v}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing N as a function of v
$N_{v\eta}$	$N_{v\eta}' = \frac{N_{v\eta}}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing N_v as a function of $(\eta - 1)$
$N_{\dot{v}}$	$N_{\dot{v}}' = \frac{N_{\dot{v}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of $\dot{v}$
N_{vq}	$N_{vq}' = \frac{N_{vq}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of the product vq
$N_{\|v\|r}$	$N_{\|v\|r}' = \frac{N_{\|v\|r}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N_r as a function of v
$N_{v\|v\|}$	$N_{v\|v\|}' = \frac{N_{v\|v\|}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing N as a function of v
$N_{v\|v\|\eta}$	$N_{v\|v\|\eta}' = \frac{N_{v\|v\|\eta}}{\frac{1}{2}\rho\ell^3}$	First order coefficient used in representing $N_{v\|v\|}$ as a function of $(\eta - 1)$

N_{vw}	$N_{vw}' = \frac{N_{vw}}{\frac{1}{2}\rho \ell^3}$	Coefficient used in representing N as a function of the product vw
N_{wp}	$N_{wp}' = \frac{N_{wp}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product wp
N_{wr}	$N_{wr}' = \frac{N_{wr}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing N as a function of the product wr
$N_{\delta r}$	$N_{\delta r}' = \frac{N_{\delta r}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing N as a function of δ_r
$N_{\delta r \eta}$	$N_{\delta r \eta}' = \frac{N_{\delta r \eta}}{\frac{1}{2}\rho \ell^3 U^2}$	First order coefficient used in representing $N_{\delta r}$ as a function of $(\eta - 1)$
p	$p' = \frac{p\ell}{U}$	Angular velocity component about y axis relative to fluid (roll)
$\dot{p}$	$\dot{p}' = \frac{\dot{p}\ell^2}{U^2}$	Angular acceleration component about x axis relative to fluid
q	$q' = \frac{q\ell}{U}$	Angular velocity component about y axis relative to fluid (pitch)
$\dot{q}$	$\dot{q}' = \frac{\dot{q}\ell^2}{U^2}$	Angular acceleration component about y axis relative to fluid
r	$r' = \frac{r\ell}{U}$	Angular velocity component about z axis relative to fluid (yaw)
$\dot{r}$	$\dot{r}' = \frac{\dot{r}\ell^2}{U^2}$	Angular acceleration component about z axis relative to fluid
U	$U' = \frac{U}{U}$	Linear velocity of origin of body axes relative to fluid
u	$u' = \frac{u}{U}$	Component of U in direction of the x axis
$\dot{u}$	$\dot{u}' = \frac{\dot{u}\ell}{U^2}$	Time rate of change of u in direction of the x axis
u_c	$u_c' = \frac{u_c}{U}$	Command speed: steady value of ahead speed component u for a given propeller rpm when body angles (α, β) and control surface angles are zero. Sign changes with propeller reversal
v	$v' = \frac{v}{U}$	Component of U in direction of the y axis
$\dot{v}$	$\dot{v}' = \frac{\dot{v}\ell}{U^2}$	Time rate of change of v in direction of the y axis

w	$w' = \frac{w}{U}$	Component of U in direction of the z axis
$\dot{w}$	$\dot{w}' = \frac{\dot{w}\ell}{U^2}$	Time rate of change of w in direction of the z axis
W	$W' = \frac{W}{\frac{1}{2}\rho\ell^2U^2}$	Weight, including water in free flooding spaces
x	$x' = \frac{x}{\ell}$	Longitudinal body axis; also the coordinate of a point relative to the origin of body axes
x_B	$x_B' = \frac{x_B}{\ell}$	The x coordinate of CB
x_G	$x_G' = \frac{x_G}{\ell}$	The x coordinate of CG
x_o	$x_o' = \frac{x_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
X	$X' = \frac{X}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along x axis (longitudinal, or axial, force)
X_{qq}	$X_{qq}' = \frac{X_{qq}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of q. First order coefficient is zero
X_{rp}	$X_{rp}' = \frac{X_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of the product rp
X_{rr}	$X_{rr}' = \frac{X_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing X as a function of r. First order coefficient is zero
$X_{\dot{u}}$	$X_{\dot{u}}' = \frac{X_{\dot{u}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of $\dot{u}$
X_{uu}	$X_{uu}' = \frac{X_{uu}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing X as a function of u in the non-propelled case. First order coefficient is zero
X_{vr}	$X_{vr}' = \frac{X_{vr}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product vr
X_{vv}	$X_{vv}' = \frac{X_{vv}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing X as a function of v. First order coefficient is zero
$X_{vv\eta}$	$X_{vv\eta}' = \frac{X_{vv\eta}}{\frac{1}{2}\rho\ell^2}$	First order coefficient used in representing X_{vv} as a function of $(\eta - 1)$
X_{wq}	$X_{wq}' = \frac{X_{wq}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of the product wq

X_{ww}	$X_{ww}' = \dfrac{X_{ww}}{\frac{1}{2}\rho \ell^2}$	Second order coefficient used in representing X as a function of w. First order coefficient is zero
$X_{ww\eta}$	$X_{ww\eta}' = \dfrac{X_{ww\eta}}{\frac{1}{2}\rho \ell^2}$	First order coefficient used in representing X_{ww} as a function of $(\eta-1)$
$X_{\delta b \delta b}$	$X_{\delta b \delta b}' = \dfrac{X_{\delta b \delta b}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_b. First order coefficient is zero
$X_{\delta r \delta r}$	$X_{\delta r \delta r}' = \dfrac{X_{\delta r \delta r}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_r. First order coefficient is zero
$X_{\delta r \delta r \eta}$	$X_{\delta r \delta r \eta}' = \dfrac{X_{\delta r \delta r \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta r \delta r}$ as a function of $(\eta-1)$
$X_{\delta s \delta s}$	$X_{\delta s \delta s}' = \dfrac{X_{\delta s \delta s}}{\frac{1}{2}\rho \ell^2 U^2}$	Second order coefficient used in representing X as a function of δ_s. First order coefficient is zero
$X_{\delta s \delta s \eta}$	$X_{\delta s \delta s \eta}' = \dfrac{X_{\delta s \delta s \eta}}{\frac{1}{2}\rho \ell^2 U^2}$	First order coefficient used in representing $X_{\delta s \delta s}$ as a function of $(\eta-1)$
y	$y' = \dfrac{y}{\ell}$	Lateral body axis; also the coordinate of a point relative to the origin of body axes
y_B	$y_B' = \dfrac{y_B}{\ell}$	The y coordinate of CB
y_G	$y_G' = \dfrac{y_G}{\ell}$	The y coordinate of CG
y_0	$y_0' = \dfrac{y_0}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axes
Y	$Y' = \dfrac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Hydrodynamic force component along y axis (lateral force)
Y_*	$Y_*' = \dfrac{Y}{\frac{1}{2}\rho \ell^2 U^2}$	Lateral force when body angles (α, β) and control surface angles are zero
Y_p	$Y_p' = \dfrac{Y_p}{\frac{1}{2}\rho \ell^3 U}$	First order coefficient used in representing Y as a function of p
$Y_{\dot p}$	$Y_{\dot p}' = \dfrac{Y_{\dot p}}{\frac{1}{2}\rho \ell^4}$	Coefficient used in representing Y as a function of $\dot p$
$Y_{p\|p\|}$	$Y_{p\|p\|}' = \dfrac{Y_{p\|p\|}}{\frac{1}{2}\rho \ell^4}$	Second order coefficient used in representing Y as a function of p

Y_{pq}	$Y_{pq}' = \frac{Y_{pq}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of the product pq
Y_{qr}	$Y_{qr}' = \frac{Y_{qr}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of the product qr
Y_r	$Y_r' = \frac{Y_r}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing Y as a function of r
$Y_{r\eta}$	$Y_{r\eta}' = \frac{Y_{r\eta}}{\frac{1}{2}\rho\ell^3 U}$	First order coefficient used in representing Y_r as a function of $(\eta-1)$
$Y_{\dot{r}}$	$Y_{\dot{r}}' = \frac{Y_{\dot{r}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of $\dot{r}$
$Y_{\|r\|\delta r}$	$Y_{\|r\|\delta r}' = \frac{Y_{\|r\|\delta r}}{\frac{1}{2}\rho\ell^3 U}$	Coefficient used in representing $Y_{\delta r}$ as a function of r
Y_v	$Y_v' = \frac{Y_v}{\frac{1}{2}\rho\ell^2 U}$	First order coefficient used in representing Y as a function of v
$Y_{v\eta}$	$Y_{v\eta}' = \frac{Y_{v\eta}}{\frac{1}{2}\rho\ell^2 U}$	First order coefficient used in representing Y_v as a function of $(\eta-1)$
$Y_{\dot{v}}$	$Y_{\dot{v}}' = \frac{Y_{\dot{v}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of $\dot{v}$
Y_{vq}	$Y_{vq}' = \frac{Y_{vq}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of the product vq
$Y_{v\|r\|}$	$Y_{v\|r\|}' = \frac{Y_{v\|r\|}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y_v as a function of r
$Y_{v\|v\|}$	$Y_{v\|v\|}' = \frac{Y_{v\|v\|}}{\frac{1}{2}\rho\ell^2}$	Second order coefficient used in representing Y as a function of v
$Y_{v\|v\|\eta}$	$Y_{v\|v\|\eta}' = \frac{Y_{v\|v\|\eta}}{\frac{1}{2}\rho\ell^2}$	First order coefficient used in representing $Y_{v\|v\|}$ as a function of $(\eta-1)$
Y_{vw}	$Y_{vw}' = \frac{Y_{vw}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of the product vw
Y_{wp}	$Y_{wp}' = \frac{Y_{wp}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of the product wp
Y_{wr}	$Y_{wr}' = \frac{Y_{wr}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of the product wr
$Y_{\delta r}$	$Y_{\delta r}' = \frac{Y_{\delta r}}{\frac{1}{2}\rho\ell^2 U^2}$	First order coefficient used in representing Y as a function of δr
$Y_{\delta r\eta}$	$Y_{\delta r\eta}' = \frac{Y_{\delta r\eta}}{\frac{1}{2}\rho\ell^2 U^2}$	First order coefficient used in representing $Y_{\delta r}$ as a function of $(\eta-1)$

z	$z' = \frac{z}{\ell}$	Normal body axis; also the coordinate of a point relative to the origin of body axes
z_B	$z_B' = \frac{z_B}{\ell}$	The z coordinate of CB
z_G	$z_G' = \frac{z_G}{\ell}$	The z coordinate of CG
z_o	$z_o' = \frac{z_o}{\ell}$	A coordinate of the displacement of CG relative to the origin of a set of fixed axe
Z	$Z' = \frac{Z}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along z axis (normal force)
Z_*	$Z_*' = \frac{Z_*}{\frac{1}{2}\rho\ell^2U^2}$	Normal force when body angles (α, [illegible]) and control surface angles are zero
Z_{pp}	$Z_{pp}' = \frac{Z_{pp}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of p. First order coefficient is zero
Z_q	$Z_q' = \frac{Z_q}{\frac{1}{2}\rho\ell^3U}$	First order coefficient used in representing Z as a function of q
$Z_{q\eta}$	$Z_{q\eta}' = \frac{Z_{q\eta}}{\frac{1}{2}\rho\ell^3U}$	First order coefficient used in representing Z_q as a function of $(\eta-1)$
$Z_{\dot{q}}$	$Z_{\dot{q}}' = \frac{Z_{\dot{q}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of $\dot{q}$
$Z_{\|q\|\delta s}$	$Z_{\|q\|\delta s}' = \frac{Z_{\|q\|\delta s}}{\frac{1}{2}\rho\ell^3U}$	Coefficient used in representing $Z_{\delta s}$ as a function of q
Z_{rp}	$Z_{rp}' = \frac{Z_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of the product rp
Z_{rr}	$Z_{rr}' = \frac{Z_{rr}}{\frac{1}{2}\rho\ell^4}$	Second order coefficient used in representing Z as a function of r. First order coefficient is zero
Z_w	$Z_w' = \frac{Z_w}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z as a function of w
$Z_{w\eta}$	$Z_{w\eta}' = \frac{Z_{w\eta}}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z_w as a function of $(\eta-1)$
$Z_{\dot{w}}$	$Z_{\dot{w}}' = \frac{Z_{\dot{w}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of $\dot{w}$
$Z_{\|w\|}$	$Z_{\|w\|}' = \frac{Z_{\|w\|}}{\frac{1}{2}\rho\ell^2U}$	First order coefficient used in representing Z as a function of w; equal to zero for symmetrical function
$Z_{w\|q\|}$	$Z_{w\|q\|}' = \frac{Z_{w\|q\|}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z_w as a function of q

$Z_{w\|w\|}$	$Z_{w\|w\|}' = \frac{Z_{w\|w\|}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing Z as a function of w
$Z_{w\|w\|\eta}$	$Z_{w\|w\|\eta}' = \frac{Z_{w\|w\|\eta}}{\frac{1}{2}\rho\ell^3}$	First order coefficient used in representing $Z_{w\|w\|}$ as a function of $(\eta-1)$
Z_{ww}	$Z_{ww}' = \frac{Z_{ww}}{\frac{1}{2}\rho\ell^3}$	Second order coefficient used in representing Z as a function of w; equal to zero for symmetrical function
$Z_{\delta b}$	$Z_{\delta b}' = \frac{Z_{\delta b}}{\frac{1}{2}\rho\ell^2 U^2}$	First order coefficient used in representing Z as a function of δ_b
$Z_{\delta s}$	$Z_{\delta s}' = \frac{Z_{\delta s}}{\frac{1}{2}\rho\ell^2 U^2}$	First order coefficient used in representing Z as a function of δ_s
$Z_{\delta s\eta}$	$Z_{\delta s\eta}' = \frac{Z_{\delta s\eta}}{\frac{1}{2}\rho\ell^2 U^2}$	First order coefficient used in representing $Z_{\delta s}$ as a function of $(\eta-1)$
α		Angle of attack
β		Angle of drift
δ_b		Deflection of bowplane or sailplane
δ_r		Deflection of rudder
δ_s		Deflection of sternplane
η		The ratio $\frac{u_c}{U}$
θ		Angle of pitch
ψ		Angle of yaw
ϕ		Angle of roll
a_i, b_i, c_i		Sets of constants used in the representation of propeller thrust in the axial equation

ABSTRACT

Standard equations of motion are presented for use in submarine simulation studies being conducted for the U.S. Navy. The equations are general enough to simulate the trajectories and responses of submarines in six degrees of freedom resulting from various types of normal maneuvers as well as for extreme maneuvers such as those associated with emergency recoveries from sternplane jam and flooding casualties. Information is also presented pertaining to the hydrodynamic coefficients and other input data needed to perform simulation studies of specific submarine designs with the Standard Equations of Motion.

ADMINISTRATIVE INFORMATION

The development of the NSRDC Standard Equations of Motion for Submarines and associated experimental techniques to provide the necessary hydrodynamic coefficients was sponsored primarily by the General Hydromechanics Research Program (Sub Project SR 009 01 01 Task 0102). Additional support was provided by the Submarine Safety Program (Sub Project S4611010 Tasks 11077 and 11083).

INTRODUCTION

The Naval Ship Research and Development Center was requested to provide the Naval Ship Engineering Center with a report on the general equations of motion currently being used for submarine simulation studies.[1] The primary purpose of the report is to establish standards, provide guidance, and establish a firm basis for contract negotiations. The report would thus serve to facilitate the Navy's dealings with contractors engaged in the manufacture of equipment such as training simulators and automatic control systems for submarines. Particular interest was expressed in having equations which would fully describe the hydrodynamic aspects of a submarine experiencing a casualty while in ahead motion, with provisions for inserting detailed representations of the casualties and recovery systems.

The Stability and Control Division of the Hydromechanics Laboratory has been conducting submarine motion simulation studies for a number of years in connection with its various assigned responsibilities. Such studies have been utilized with considerable success to solve a wide variety of problems pertaining to the design and operation of submarines from the standpoint of stability and control. Most recently, these studies have included: emergency recovery from sternplane jams, load supportability, and vertical ascents after emergency blowing (both with and without forward propulsion). These simulation studies have been made possible as the result of a progressive in-house development of

[1] References are listed on page 27.

six-degree-of-freedom motion equations and a coordinated development of advanced experimental and analytical techniques to provide the necessary coefficients.

The equations of motion as they now exist at NSRDC are general enough to simulate trajectories of submarines for all of the various types of normal maneuvers and emergency situations that have been of concern up to the present time. Furthermore, it is believed that they will adequately cover most of the situations that may be reasonably anticipated for the future. These equations, which form the subject of this report, have been used continuously for the past three years with little change, and are considered to be standard at NSRDC. In the interest of consistency, it is proposed herein that these equations be adopted and serve as the U.S. Navy Standard until notice is given by NSRDC that they should be changed. It should be emphasized, however, that these equations have been validated only with the types of coefficients being generated by the Hydromechanics Laboratory. Therefore, prospective users are advised to first contact NSRDC to determine whether the required hydrodynamic coefficients are available.

It is the intention eventually to issue a complete and comprehensive report on the NSRDC Standard Equations of Motion for Submarines which would be tantamount to a textbook. This would include a discussion of the physics involved and the various means for determining the numerical values of each of the individual hydrodynamic coefficients required for the equations. In addition, it is planned to discuss the various computer techniques used for simulation and to present those computer programs which have reached an advanced state of development. Such a treatise, however, is considered to be much beyond the scope of what is required to satisfy the immediate needs stated in the request.

To accomplish the specific objectives mentioned at the outset, this report presents a brief history of the development of the equations of motion at NSRDC; defines the nomenclature, axes systems, and sign conventions used for the equations; presents the NSRDC Standard Equations of Motion for conducting submarine simulation studies; outlines the methods and sources for obtaining the hydrodynamic coefficients and other input data required for simulation; and briefly discusses the application and range of validity of the Standard Equations.

HISTORY

The equations of motion now in use at NSRDC are the direct result of a continuing program of in-house research carried out by the Stability and Control Division of the Hydromechanics Laboratory. This program was officially started in June 1956 when it became evident that a rapid technological expansion was taking place in the area of submarine stability and control. The decision was made at that time to develop general equations of motion for six degrees of freedom which could be used in conjunction with coefficients obtained from model experiments to predict and study the behavior of submarines and associated systems

in a variety of maneuvers. Concurrently, studies were undertaken to develop suitable techniques to determine experimentally all of the hydrodynamic coefficients required for these equations for any arbitrary submerged body-appendage configuration. The Planar-Motion-Mechanism System, which was constructed and placed into regular service in 1957, was one outgrowth of these studies. This system is still the standard method used at NSRDC to determine numerically most of the required coefficients. It was realized at the outset that many years of research would be required to develop, implement, and validate a completely general set of equations that could be used to study all situations involved in the motions of submerged bodies. In view of the pressing needs, however, it was decided to approach the ultimate objective by progressive stages.

The first effort was directed toward the development of equations to permit simulation of normal types of maneuvers involving six degrees of freedom, such as those required by submarines to fulfill effectively their various missions. The existing linearized equations of motion were first used as a basis to study moderate or small-scale maneuvers. These were later extended to include various nonlinear and coupling terms. By 1960, it was felt that the state of the art had progressed to the point where some standardization would be desirable. Accordingly, the Stability and Control Division of the Hydromechanics Laboratory adopted a "standard" set of six-degree-of-freedom equations for submarines in normal types of maneuvers. The term standard was applied in the sense that the equations were to be used for their designated purpose in all studies made by the Laboratory, and were to have official status until a sufficient advance was made in the state of the art to justify a change. If such turned out to be the case, it was intended that the revisions would be made officially and a new set of standard equations would be issued.

Together with the adoption of standard equations, a continuing program was initiated to correlate full-scale trial measurements with computer predictions involving these equations. On basis of correlation studies made subsequently, it appeared that these standard equations together with coefficients determined with the Planar-Motion-Mechanism System would, for most part, yield accurate predictions of the submarine trajectories for a variety of normal or definitive maneuvers that could be used to evaluate the handling qualities of submarines.

During 1962, in anticipation of a program directed toward the development of control procedures and auxiliary devices to improve the emergency recovery capabilities of submarines, the research program was reoriented to establish prediction techniques for studying the behavior of submarines in a variety of possible emergency situations. The revised research program included fundamental theoretical and experimental studies such as: expanding the standard equations of motion to include terms (such as those associated with "backing" on propellers and blowing of ballast tanks) which were not required for simulation of normal maneuvers; extending the range of validity of hydrodynamic coefficients to provide accurate representations in extreme situations (such as those involving angles of attack up to 90 degrees); and establishing correlation

between predicted trajectories and those obtained on full-scale submarines during "emergency recovery trials".

About December 1963, the Bureau of Ships requested the Laboratory to consider undertaking a program of model tests and simulation studies to determine the ability of each of twenty-two different submarine types to recover from sternplane jam emergencies. By this time, considerable progress pertinent to the sternplane jam problem had been made on the development of the equations of motion. Therefore, the Laboratory agreed to accept full responsibility for carrying out the proposed program, provided the simulation studies could be carried out with equations that constituted the "state of the art" as of February 1964.

In July 1964, at the request of the Bureau, the Laboratory extended its program to include load supportability studies on the various submarine types. This study was well within the scope of existing equations and consequently no further development was required. However, in September 1964, when the problem of roll associated with free rise of submarines after blowing was introduced, it became necessary to develop new model-test techniques and to conduct further analytical studies to extend the equations to permit detailed simulation studies of the problem. Meanwhile, as the result of research studies on the Rotating Arm Facility, further refinements were made in certain coupling terms associated with "squatting" behavior while turning, and these were incorporated into the equations. Thus, the equations contained in this report represent the state of the art which has existed since about March 1965.

STANDARD EQUATIONS OF MOTION

The NSRDC Standard Equations of Motion for Submarine Simulation, briefly designated herein as the Standard Equations, are presented in their entirety in this section to provide a means of ready reference. The Standard Equations are sufficiently comprehensive to define the trajectories and responses, in six degrees of freedom, of a submarine undergoing even extreme maneuvers such as an emergency recovery from a hard-dive sternplane jam, including the buoyant ascent after blowing of ballast. Ordinarily, the Standard Equations are used for ahead motion; however, by pertinent changes in the coefficients they can be used for separate studies of astern motions.

A complete notation for the various terms in the equations is given in the front of the report. This notation is generally in accordance with the Standard Nomenclature given in References 2 and 3. The Standard Equations are referred to a right-hand orthogonal system of moving axes, fixed in the body, with its origin located at the center of mass CG of the body. The xz plane is the principal plane of symmetry (vertical centerplane for submarines); the x axis is parallel to the baseline of the body. The positive directions of the axes are specified as follows: x-forward, y-starboard, and z-downward. The remaining sign conventions follow from the right-hand-screw rule. The positive directions of the axes, angles, linear velocity components, angular velocity components, forces, and moments are shown by arrows in the sketch of Figure 1.

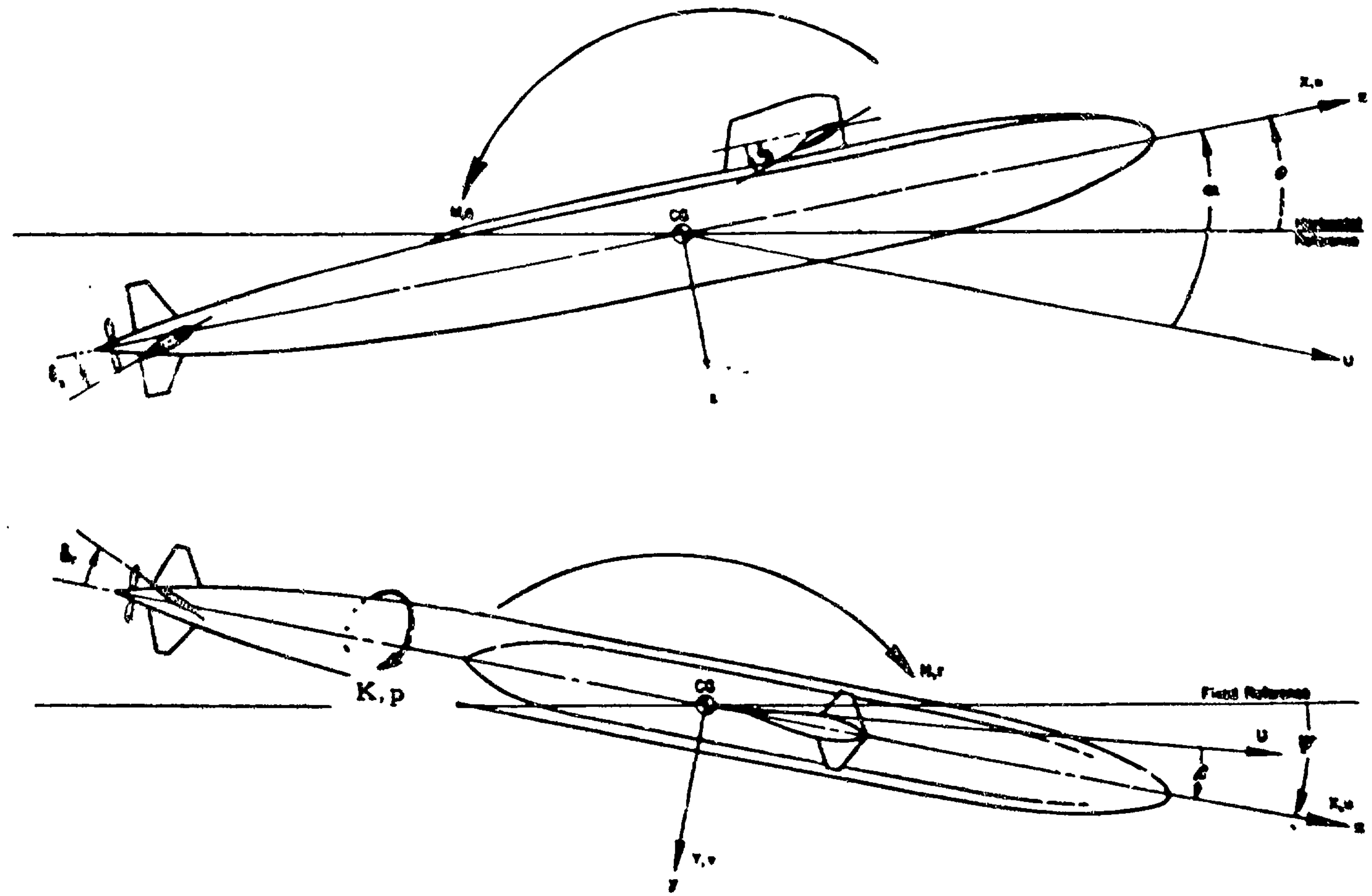

Figure 1 - Sketch Showing Positive Directions of Axes, Angles, Velocities, Forces, and Moments

The equations are presented in the following order: axial force, lateral force, normal force, rolling moment, pitching moment, and yawing moment. In addition certain kinematic relations are given.

AXIAL FORCE

$$m\left[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2} \ell^4 \left[X_{qq}{}' q^2 + X_{rr}{}' r^2 + X_{rp}{}' rp\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}{}' \dot{u} + X_{vr}{}' vr + X_{wq}{}' wq\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{uu}{}' u^2 + X_{vv}{}' v^2 + X_{ww}{}' w^2\right]$$

$$+ \frac{\rho}{2} \ell^2 u^2 \left[X_{\delta r\,\delta r}{}' \delta r^2 + X_{\delta s\,\delta s}{}' \delta s^2 + X_{\delta b\,\delta b}{}' \delta b^2\right]$$

$$+ \tfrac{1}{2} \rho \ell^2 \left[a_i u^2 + b_i u u_c + c_i u_c^2\right]$$

$$- (W - B) \sin\theta$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{vv\eta}{}' v^2 + X_{ww\eta}{}' w^2 + X_{\delta r \delta r \eta}{}' \delta_r^2 u^2\right.$$

$$\left. + X_{\delta s \delta s \eta} \delta_s^2 u^2\right](\eta - 1)$$

LATERAL FORCE

$$m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$

$$+\frac{\rho}{2}\ell^4\left[Y_{\dot{r}}'\dot{r} + Y_{\dot{p}}'\dot{p} + Y_{p|p|}'p|p| + Y_{pq}'pq + Y_{qr}'qr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_{\dot{v}}'\dot{v} + Y_{vq}'vq + Y_{wp}'wp + Y_{wr}'wr\right]$$

$$+\frac{\rho}{2}\ell^3\left[Y_r'ur + Y_p'up + Y_{|r|\delta r}'u|r|\delta r + Y_{v|r|}'\frac{v}{|v|}\left|(v^2 + w^2)^{\frac{1}{2}}\right||r|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_*'u^2 + Y_v'uv + Y_{v|v|}'v\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\ell^2\left[Y_{vw}'vw + Y_{\delta r}'u^2\delta r\right]$$

$$+(W - B)\cos\theta\sin\phi$$

$$+\frac{\rho}{2}\ell^3 Y_{r\eta}'ur(\eta-1)$$

$$+\frac{\rho}{2}\ell^2\left[Y_{v\eta}'uv + Y_{v|v|\eta}'v\left|(V^2 + w^2)^{\frac{1}{2}}\right| + Y_{\delta r\eta}'\delta_r u^2\right](\eta-1)$$

NORMAL FORCE

$$m\left[\dot{w} - uq + vp - z_G\,(p^2 + q^2) + x_G\,(rp - \dot{q}) + y_G\,(rq + \dot{p})\right] =$$

$$+\frac{\rho}{2}\,\ell^4\left[Z_{\dot{q}}'\,\dot{q} + Z_{pp}'\,p^2 + Z_{rr}'\,r^2 + Z_{rp}'\,rp\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_{\dot{w}}'\,\dot{w} + Z_{vr}'\,vr + Z_{vp}'\,vp\right]$$

$$+\frac{\rho}{2}\,\ell^3\left[Z_q'\,uq + Z_{|q|\delta s}'\,u|q|\delta s + Z_{w|q|}'\,\frac{w}{|w|}\left|(v^2 + w^2)^{\frac{1}{2}}\right|\,|q|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_*'\,u^2 + Z_w'\,uw + Z_{w|w|}'\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{|w|}'\,u|w| + Z_{ww}'\,\left|w\,(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{vv}'\,v^2 + Z_{\delta s}'\,u^2\,\delta s + Z_{\delta b}'\,u^2\,\delta b\right]$$

$$+ (W - B)\cos\theta\cos\phi$$

$$+\frac{\rho}{2}\,\ell^3\,Z_{q\eta}'\,uq\,(\eta - 1)$$

$$+\frac{\rho}{2}\,\ell^2\left[Z_{w\eta}'\,uw + Z_{w|w|\eta}'\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right| + Z_{\delta s\eta}'\,\delta_s u^2\right](\eta - 1)$$

ROLLING MOMENT

$$I_x \dot{p} + (I_z - I_y)\, qr - (\dot{r} + pq)\, I_{xz} + (r^2 - q^2)\, I_{yz} + (pr - \dot{q})\, I_{xy}$$

$$+ m \Big[y_G (\dot{w} - uq + vp) - z_G (\dot{v} - wp + ur) \Big] =$$

$$+ \frac{\rho}{2} \ell^5 \Big[K_{\dot{p}}' \dot{p} + K_{\dot{r}}' \dot{r} + K_{qr}' qr + K_{pq}' pq + K_{p|p|}' p|p| \Big]$$

$$+ \frac{\rho}{2} \ell^4 \Big[K_p' up + K_r' ur + K_{\dot{v}}' \dot{v} \Big]$$

$$+ \frac{\rho}{2} \ell^4 \Big[K_{vq}' vq + K_{wp}' wp + K_{wr}' wr \Big]$$

$$+ \frac{\rho}{2} \ell^3 \Big[K_*' u^2 + K_v' uv + K_{v|v|}' v \big|(v^2 + w^2)^{\frac{1}{2}}\big| \Big]$$

$$+ \frac{\rho}{2} \ell^3 \Big[K_{vw}' vw + K_{\delta r}' u^2 \delta r \Big]$$

$$+ (y_G W - y_B B) \cos\theta \cos\phi - (z_G W - z_B B) \cos\theta \sin\phi \Big]$$

$$+ \frac{\rho}{2} \ell^3 K_{*\eta}' u^2 (\eta - 1)$$

PITCHING MOMENT

$$I_y \dot{q} + (I_x - I_z)\, rp - (\dot{p} + qr)\, I_{xy} + (p^2 - r^2)\, I_{zx} + (qp - \dot{r})\, I_{yz}$$

$$+ m \left[z_G (\dot{u} - vr + wq) - x_G (\dot{w} - uq + vp) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[M_{\dot{q}}' \dot{q} + M_{pp}' p^2 + M_{rr}' r^2 + M_{rp}' rp + M_{q|q|}' q|q| \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_{\dot{w}}' \dot{w} + M_{vr}' vr + M_{vp}' vp \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_q' uq + M_{|q|\delta s}' u|q|\delta s + M_{|w|q}' \left|(v^2 + w^2)^{\frac{1}{2}}\right| q \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_*' u^2 + M_w' uw + M_{w|w|}' w \left|(v^2 + w^2)^{\frac{1}{2}}\right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{|w|}' u|w| + M_{ww}' \left|w (v^2 + w^2)^{\frac{1}{2}}\right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{vv}' v^2 + M_{\delta s}' u^2 \delta s + M_{\delta b}' u^2 \delta b \right]$$

$$- (x_G W - x_B B) \cos\theta \cos\phi - (z_G W - z_B B) \sin\theta$$

$$+ \frac{\rho}{2} \ell^4 M_{q\eta}' uq (\eta - 1)$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{w\eta}' uw + M_{w|w|\eta}' w \left|(v^2 + w^2)^{\frac{1}{2}}\right| + M_{\delta s\eta}' \delta_s u^2 \right] (\eta - 1)$$

YAWING MOMENT

$$I_z \dot{r} + (I_y - I_x)\, pq - (\dot{q} + rp)\, I_{yz} + (q^2 - p^2)\, I_{xy} + (rq - \dot{p})\, I_{zx}$$

$$+ m \left[x_G (\dot{v} - wp + ur) - y_G (\dot{u} - vr + wq) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[N_{\dot{r}}' \dot{r} + N_{\dot{p}}' \dot{p} + N_{pq}' pq + N_{qr}' qr + N_{r|r|}' r|r| \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[N_{\dot{v}}' \dot{v} + N_{wr}' wr + N_{wp}' wp + N_{vq}' vq \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[N_p' up + N_r' ur + N_{|r|\delta r}' u|r|\delta r + N_{|v|r}' \left|(v^2 + w^2)^{\frac{1}{2}}\right| r \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[N_*' u^2 + N_v' uv + N_{v|v|}' v \left|(v^2 + w^2)^{\frac{1}{2}}\right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[N_{vw}' vw + N_{\delta r}' u^2 \delta r \right]$$

$$+ (x_G W - x_B B) \cos\theta \sin\phi + (y_G W - y_B B) \sin\theta$$

$$+ \frac{\rho}{2} \ell^4 N_{r\eta}' ur (\eta - 1)$$

$$+ \frac{\rho}{2} \ell^3 \left[N_{v\eta}' uv + N_{v|v|\eta}' v \left|(v^2 + w^2)^{\frac{1}{2}}\right| + N_{\delta r\eta}' \delta_r u^2 \right] (\eta - 1)$$

KINEMATIC RELATIONS

$$U^2 = u^2 + v^2 + w^2$$

$$\dot{z}_o = -u \sin\theta + v \cos\theta \sin\phi + w \cos\theta \cos\phi$$

$$\dot{\phi} = p + \dot{\psi} \sin\theta$$

$$\dot{\theta} = \frac{q - \dot{\psi} \cos\theta \sin\phi}{\cos\phi}$$

$$\dot{\psi} = \frac{r + \theta \sin\phi}{\cos\theta \cos\phi}$$

IDENTITY AND SOURCE OF INPUT DATA

As mentioned in the Introduction, it is beyond the scope of this report to go into a detailed treatment on the use of computer techniques to perform simulation studies involving the Standard Equations. The purpose of this section is merely to identify the kinds of input data required to perform such studies and to indicate where such data can be obtained. In addition, an attempt is made to indicate some of the sub-routines that are also needed to perform various types of simulation studies. In particular, those sub-routines required for the sternplane jam phase of the Submarine Safety Program are listed. A somewhat more detailed account of the nature of the hydrodynamic coefficients and the processes for obtaining them is relegated to the next section.

The NSRDC Standard Equations of Motion given in the preceding section are written in a form utilizing nondimensional coefficients, and are applicable in a general sense to the rigid body motions of submarines and other submerged vehicles. Certain input data are required, however, to obtain a mathematical model which can be used to realistically simulate the motions of a specific submarine design. A complete set of input data for this purpose consists of the numerical values of all of the hydrodynamic coefficients, inertial properties, and pertinent geometric characteristics for the given submarine that enter into the various terms of the equations. Fortunately, as the result of the concentrated effort on the Submarine Safety Program, as well as contract design studies made prior to construction, complete sets of input data for about 25 different existing submarine designs are on hand at NSRDC. The data, for most part, carry the classification of "CONFIDENTIAL". However upon request, they can be furnished to outside agencies and contractors having the proper security clearance and "need to know".

Similar data can be obtained for new submarine designs by addressing a request to the Commanding Officer and Director of the Naval Ship Research and Development Center. This request should be accompanied by an outline of program objectives, lines plans, and other pertinent data. An estimate of time and cost will be provided, after which funds must be deposited at NSRDC before work can proceed. If the information is needed on a short term basis, this should be clearly stated by the requesting agency so that sufficient lead time, particularly to allow for model construction and scheduling of test facilities, can be allowed. Furthermore, in contractual dealings, an early commitment to supply the data should be obtained from NSRDC before final negotiations are made. This is especially stressed for those contracts in which time is of the essence, or where the contractor may understand that having prior possession of the information is a condition precedent to performance.

The primary mathematical model, consisting of the Standard Equations and input data pertinent to a specific submarine, can be readily programmed on either an analog or digital computer. To perform a complete motion simulation study, however, the computer program must

also include certain supplementary mathematical models or sub-routines. For example, to conduct a complete simulation study of the motions in six degrees of freedom of a submarine involved in emergency recovery from a sternplane jam casualty requires at least the sub-routines which include the following representations:

1. Time history of sternplane movement in degrees including various prescribed sternplane "jams"

2. Time history of sailplane movement in degrees starting from time the order is given

3. Time history of rudder movement in degrees starting from time the order is given with provision for various modes of motion such as swinging the rudder sharply over to a hardover position (either right or left), or fishtailing

4. Time history of change in propeller rpm from "full ahead" to "back emergency" starting from the time the order is given and including appropriate time allowances for communication, human reaction and manipulation of controls, and prime-mover response

5. Time history of weight of ballast water discarged or taken on due to blowing or venting, respectively each of the main ballast tanks. The time is measured from the instant the order is given and includes the effects of system lags, as well as pressure changes due to changes in air-bank pressure and submarine depth on rate of water discharge.

The first three of these sub-routines convert time histories of control surface movement to time histories of hydrodynamic forces and moments exerted on the submarine hull when they are applied to the terms in the primary mathematical model containing δ_s, δ_r, and δ_b. The time history of propeller rpm is converted to external hydrodynamic forces and moments by means of the terms containing $(\eta-1)$ and u_c. These terms reflect the incremental changes in forces and moments due to either over or under propulsion (including backing on the propellers while the submarine is proceeding ahead). For the moderate changes in ahead speed involved in most normal maneuvers, all of the $(\eta-1)$ terms usually can be neglected. The sub-routine for blowing or venting of ballast tanks is already in the form of forces and moments. It can be tied in directly with the primary mathematical model by replacing the constant weight terms, such as W-B, with the appropriate time histories of weight changes.

It is apparent that the primary mathematical model, in conjunction with the foregoing types of sub-routines, can be used to simulate the submarine undergoing a host of other normal submerged maneuvers as well. These maneuvers, however, are essentially of the open loop type. To perform closed loop studies, such as depthkeeping or coursekeeping under the influence of various environmental conditions, other sub-routines

including either manual operators or automatic control systems must be added. A typical simulation study of this type is the problem of maintaining periscope depth while operating under a heavy sea (say State 5). A sub-routine to represent the effects of an ahead, random sea on the submarine has been developed and used by NSRDC on a number of occasions in the past. This sub-routine, sometimes called the sea-state analog, is in the form of forcing functions which are generally applied in an analog computer simulation by means of a white-noise generator, the output of which, in some cases, has been prerecorded on magnetic tape. The major components of the sea-state analog are as follows:

1. The surface configuration of the representative random seaway
2. The effects of the seaway on the depth (pressure) sensor which are functions of depth
3. The oscillatory forces and moments acting on the submarine hull which are functions of depth
4. The suction forces acting on the submarine which are functions of depth

The Standard Equations have been programmed at NSRDC both on the analog computers of the Systems Simulation Facility of the Hydromechanics Laboratory and the digital computer (IBM 7090) of the Applied Mathematics Laboratory. These programs, however, have not yet been adequately documented for general publication. Similarly, computer programs exist for a variety of sub-routines. Since these sub-routines usually must be tailored to a particular problem, or in some cases to a particular computer, no concerted effort has been made to systematize and document them for general publication. Nevertheless, the input data required to construct the sub-routines for existing submarines which have been studied are available at NSRDC and can be furnished to those entitled to receive them. Other sources for such input data, particularly those for sub-routines dealing with hardware items such as propulsion machinery, ballast blowing systems, control linkages, and automatic control systems are: the Naval Ship Engineering Center (NAVSEC), the submarine construction shipyards, and the equipment manufacturers.

THE HYDRODYNAMIC COEFFICIENTS

The hydrodynamic coefficients constitute the heart of the primary mathematical model used in simulation studies of rigid-body motions of submarines. In fact, the form selected for the equations of motions is influenced to a large extent by the kinds of hydrodynamic coefficients employed. With this qualification in mind, this section of the report presents further information as to the nature of the hydrodynamic coefficients used at NSRDC as well as some insight into how they are obtained. It is hoped that this will serve to assure that the hydrodynamic coefficients being used for a particular study are compatible with the Standard Equations.

Unless mentioned otherwise, the hydrodynamic coefficients apply to the deeply submerged case, free of free-surface, bottom, and wall effects.

The hydrodynamic forces and moments which enter into the Standard Equations as coefficients are usually classified into three general categories: static, rotary, and acceleration. The static coefficients are due to the components of linear velocity of the body relative to the fluid; the rotary coefficients are due to components of angular velocity; and the acceleration coefficients are due to either linear or angular acceleration components. Within limited ranges, the coefficients are linear with respect to the appropriate variables, and thus may be utilized as static, rotary, and acceleration derivatives in linearized equations of motion.

For six degrees of freedom, the coefficients in each of the three categories which appear in the Standard Equations are quite numerous. The problem thus becomes one of determining the numerical values of the individual coefficients with sufficient accuracy to support the objectives of the particular simulation study involved. Ideally, it would be desirable to acquire the required numerical values for a given submarine configuration by means of hydrodynamic theory. Unfortunately, those coefficients which are primarily due to viscous flow, such as the static and rotary coefficients, cannot be obtained reliably using only existing theory. Theory has been used with reasonable success to compute acceleration coefficients for simple forms without appendages, such as bodies of revolution, which are amenable to treatment on basis of potential flow considerations. However, for actual submarine configurations which include appendages such as control surfaces, decks, bridge fairwaters, and propellers, the use of theory to determine the values for coefficients even of this type with the accuracy required for certain simulation studies becomes somewhat questionable. Accordingly, the present state of the art is to rely almost entirely on experimental means for determining the numerical values of the hydrodynamic coefficients for specific design problems.

The primary methodology used at NSRDC to obtain the required hydrodynamic coefficients is the Planar-Motion-Mechanism System[5]. This system was conceived and developed in the Stability and Control Division of the Hydromechanics Laboratory. It incorporates in one device a means for experimentally determining all of the types of hydrodynamic coefficients in each of the three categories that are required in equations of motion for a submerged body in six degrees of freedom. A complete description of the system including its basic concepts, principles of operation, apparatus, instrumentation, and typical test results, is given in References 5 and 6. Planar-Motion-Mechanism System, Mark I, as it is presently constituted, can be used to obtain the numerical values of all of the coefficients except certain coupling terms and those nonlinearities associated with high values of nondimensional angular velocity component, such as r', which are associated with maneuvers involving relatively tight turns. Where such values are required, the data obtained from Planar-Motion-Mechanism tests are supplemented by the results of Rotating-Arm tests and/or estimates based on theory. The Rotating Arm Facility and the instrumentation and techniques used to conduct submarine

model tests thereon are described in References 7 and 6, respectively. Further details on the experimental and estimation techniques which relate to the kinds and quality of coefficients to be used specifically with the Standard Equations are presented in the following paragraphs.

The Standard Equations are written in terms of the complete submarine configuration. Accordingly, for quantitative studies pertaining to specific submarine designs, the standard practice at NSRDC is to conduct all of the required tests with models that are fully equipped with all significant appendages including bridge fairwater, deck, bowplanes or sailplanes, sternplanes, rudders and propellers. These tests effectively cover a matrix containing a range of motion variables (referred to the hull center of gravity) that generally exceeds that which could be encountered by the prototype submarine, and ranges of control surface angles and propeller rpm's which are at least equal to the capacity of the specified submarine. The resulting characterization of hydrodynamic forces and moments thus embraces interaction effects involved in the various modes of rigid body motion between: control surfaces and hull, propeller and hull, and propeller and stern control surfaces. Also included are downwash effects of forward appendages, such as bridge fairwater, sailplanes, and deck, acting on the stern control surfaces.

Since the foregoing process yields hydrodynamic coefficients which pertain to the complete configuration, there is no need to include separate terms in the equations in an attempt to account for local angles of attack in way of control surfaces. The latter procedure has been suggested by some other investigators as a means for obtaining a better representation of control-surface stall, particularly where large local angles of attack occur when the control surfaces are deflected. On the other hand, the procedure for testing the complete configuration embraces the effects on control coefficients due to changes in local angles of attack resulting from the various modes of motion. Since true flow angles of attack are inherent in this process, the resulting control coefficients should provide a more accurate representation. In fact, if the two procedures are combined, and numerical values are assigned to the various coefficients, there is always a possibility of creating a redundancy in the equations. It is strongly recommended, therefore, if a complete set of hydrodynamic coefficients, based primarily on experiment is furnished by NSRDC, that the numerical values of all coefficients be used directly without alteration.

In addition to their being completely equipped and self-propelled, as a matter of standard practice, the models used at NSRDC for tests involving specific submarine designs are large (usually about 20 feet in length). Such large models used in conjunction with the large, rigid towing tank facilities of the Hydromechanics Laboratory permit the determination of hydrodynamic coefficients which are comparatively free of scale effects and other extraneous experimental problems. The advantages of large models and large facilities are particularly prevalent when obtaining data at high angles of attack on the hull and/or control surfaces. It is essential, in both cases, that the Reynolds number $\frac{U\ell}{\nu}$ be high enough to

avoid the effects of transitional flow over the hull and appendages of the model. The effects of transitional flow at large hull angles of attack are manifested as cross-flow drag coefficients that are much higher than they should be for the corresponding full-scale submarine. The effect of transitional flow on the control surfaces is primarily manifested as premature stall (breakdown in forces) on the model control surfaces. This results in control coefficients at the larger control deflections which are lower than they should be for the corresponding full-scale submarine. With the large models, a sufficiently high Reynolds number can be obtained at a moderate speed, say about 6 knots for a 20-foot model, to avoid scale effects in most cases. With the large facilities, i.e. facilities having cross-sectional dimensions that are large compared with the corresponding dimension of the model, these Reynolds numbers can be obtained without encountering blockage or free-surface effects which require corrections to the data. Other advantages in the use of large models include: minimization of tow-strut interference effects,[5] reduction of propeller scale effects, ease of model alignment, accuracy of model ballasting, and ability to house standard instrumentation, propulsion motors, and control actuators within the model. All of these factors contribute to the accuracy and repeatability of the end results.

It should be noted that the use of small models (5 feet long or less) does not in itself preclude the ability to obtain hydrodynamic force and moment data which, for the deep submergence case, are essentially free of transitional flow effects. For example, in water of the same temperature, the same Reynolds number can be obtained with a 5-foot model at a speed of 24 knots as that mentioned as adequate for the 20-foot model at 6 knots. Unfortunately, most of the laboratories which employ small models have relatively small facilities especially if they have the speed capability. Therefore, the blockage effects constitute the limiting problem, particularly when conducting tests at large hull angles of attack.

Where fairly complete data are available for a specific submarine design as the result of an extensive series of tests with the Planar-Motion-Mechanism System, the decision frequently is made to forego supplementary Rotating-Arm tests to obtain the missing nonlinearities mentioned previously. This is because estimation techniques, based on empirical data derived from prior Rotating-Arm tests with comparable submerged bodies, have been developed by NSRDC. Indications to date are that these estimation techniques are sufficiently accurate for the purpose of making nearly all of the various kinds of simulation studies required for most modern types of military submarines. Therefore, in these cases, it is felt that any possible further refinement would not justify the costs of the supplementary tests with the Rotating-Arm Facility. Also, as mentioned previously, there are a few coupling coefficients that are not being presently obtained by either of the two experimental techniques. These are estimated on basis of theory using the relationships given in Reference 8.

The hydrodynamic data required for simulation studies of a specific submarine design are usually transmitted by NSRDC to the users in a standardized tabular form. The table contains a complete listing of the numerical values for each of the coefficients shown in the Standard Equations. The numerical values of the coefficients are derived as follows:

The hydrodynamic force and moment data obtained by the aforementioned experimental and analytical techniques are first reduced to nondimensional form in accordance with the normalization formulas given in the Notation. These data can then be plotted or tabulated as functions of each of the appropriate nondimensional kinematic variables. In general, a least square fit can then be applied to the data representing each of the functional relationships to obtain the desired nondimensional coefficients. Certain functional relationships are known to be linear from considerations of theory, such as those associated with the acceleration or "added mass" coefficients. In such cases, only a straightline fit is made, and the resulting numerical values can be used interchangeably as coefficients in the Standard Equations or as stability derivatives in linearized equations of motion. For nonlinear functional relationships, least square fits to polynomials are used. The current practice, in nearly all cases, is to carry such fits only up to the second order coefficients. This is done primarily to facilitate computer representation, particularly where analog computer techniques are to be employed. The use of a second order representation, however, has at least some foundation in theory for certain quantities such as those arising from cross-flow drag. It should be emphasized that the numerical values of first order coefficients associated with second order fits are not necessarily equal to those customarily used for corresponding stability and control derivatives in linearized equations of motion. However, the standard notation used to describe the various first order coefficients, including the acceleration coefficients previously mentioned, is the same as that used for the corresponding stability and control derivatives.[3] To avoid misunderstanding, therefore, all of the values contained in the standardized tables provided by NSRDC are to be taken as coefficients of the Standard Equations to be used for simulation studies, unless mentioned otherwise. Any table which contains numerical values being provided for use in stability and control analyses involving linearized equations of motion, will be clearly marked as a table of stability and control derivatives.

It may be noted in the Standard Equations that some of the coefficients involve absolute values of certain kinematic variables. This is done to assure the proper signs in the computer representations and, in some cases, to obtain a better representation when a function happens to be asymmetrical, such as the variation of M' with w' with respect to rise and dive.

Although the standardized tables list all of the coefficients shown in the Standard Equations, some of the coefficients may be assigned a value of zero. This is because, for some submarine types, the values are actually zero or small enough to be neglected. Furthermore, for certain submarine types, it may not be necessary to represent the coefficients that vary with η when studying only normal types of maneuvers.

In addition to the hydrodynamic coefficients shown explicitly in the Standard Equations for the deep submergence case, NSRDC provides other hydrodynamic data for use in submarine simulation studies. Typical data in this respect are the coefficients or forcing functions associated with proximity to: the free surface (either in still water or waves), the ocean bottom, or other boundaries. These data are obtained by model experiment, by theory, or by a combination of both.

An example of such supplementary data are the hydrodynamic forcing functions used to construct the sea-state analog which was mentioned earlier as one of the sub-routines used for depthkeeping studies. These forcing functions are compounded on the basis of the forces and moments acting on th submerged body moving under trains of regular waves of progressively varied frequencies and amplitudes. When a body moves at constant speed beneath a regular train of waves of a given period and amplitude, two general types of forces arise: oscillatory forces having a frequency equivalent to the encounter frequency, and a constant force called a suction force which always tends to pull the submarine toward the surface. The oscillatory forces and moments are determined experimentally, in some cases. This is done by conducting special tests in which the model is restrained to the towing carriage at each of several depths near the surface and towed over a range of constant speeds for each of several uniform wave conditions. The resulting forces and moments and the wave configuration are recorded as time histories. The phase angle between the forces and moments and wave, referred to the hull center of gravity, is also determined. Oscillatory forces and moments which, in many cases, are sufficiently accurate for depthkeeping studies can be determined by the method of Reference 9. The suction forces are difficult to determine experimentally. Therefore, the usual practice is to compute them by means of theory.[10]

Once the oscillatory and suction forces have been determined for a range of cases involving regular waves, they are compounded into spectra for the case of a representative random sea, say a State 5 sea, by a technique developed by the Hydromechanics Laboratory. It should be mentioned that although the suction forces generated by this technique are random, they always remain directed toward the surface. The data for the seaway configuration, the forcing functions, and response of the depth sensor to the seaway are presented in spectral form, including functions to account for attenuation with increase in depth. In some cases, the data have been recorded on magnetic tapes for runs of 20-minute duration which can be used directly in an analog computer simulation.

APPLICATION AND RANGE OF VALIDITY

The NSRDC Standard Equations of Motion can be used in conjunction with the aforementioned hydrodynamic coefficients and sub-routines to perform a wide variety of submarine simulation studies. Several of these applications have been mentioned earlier in a somewhat different context.

The purpose of this section is to present a more complete listing of the various applications and discuss their ranges of validity.

Simulation techniques, such as those discussed in this report, have progressed to the point where they have become the primary tools for studying rigid-body motions and related phenomena pertaining to submarines. As mentioned in the Introduction, such techniques already have been applied to solve a wide variety of problems pertaining to the design and operation of submarines. In addition, they have been used effectively with training simulators to achieve improved Fleet readiness, with large savings in personnel-training costs. Undoubtedly, there are numerous other applications which will become apparent as time goes by. The following is a compilation, not necessarily all inclusive, of the various categories of simulation studies that have been carried out using the Standard Equations. Unless mentioned otherwise, all of the maneuvers involved in these studies pertain to the submerged condition in ahead motion.

1. Definitive maneuvers (open loop) to evaluate inherent handling qualities
   a. Meanders (vertical plane)
   b. Vertical overshoots
   c. Horizontal steady turns
   d. Horizontal overshoots
   e. Horizontal spirals
   f. Acceleration and deceleration in straightline motion

2. Normal maneuvers (operational or tactical)
   a. Depthkeeping and coursekeeping at various speeds, including hovering, using manual or automatic control
      1. Deeply submerged with environmental disturbances such as density gradients and cross-currents
      2. Near-surface under various sea states
      3. Near-bottom including large-scale bottom irregularities
   b. Limit dives using manual, semi-automatic, or automatic control
   c. Transient horizontal turns using manual, semi-automatic, or automatic control
   d. Spiral descents
   e. Mission profiles of various types including target tracking, weapons delivery, and a variety of evasive maneuvers

3. Emergency Maneuvers
   a. Recovery from sternplane jam casualties using various combinations of recovery measures

b. Recovery from flooding casualties using various combinations of recovery measures

c. Buoyant ascents to develop safe procedures for exercising emergency ballast blow systems

d. Maneuvers to determine load supportability as a function of speed

The main strength of the simulation techniques described herein lies in their ability to predict or represent, with reasonable accuracy, the behavior of a given submarine in a variety of modes of motion. Furthermore, if maximum benefit is to be derived, these techniques should perform this function either in advance of construction, or without involvement of the full-scale submarine where it already exists. To arrive at this state of the art, it is apparent that an active and vigilant correlation program to verify the accuracy and validity of predictions must be maintained.

The Hydromechanics Laboratory devotes a large part of its efforts to the business of prediction, namely providing solutions to problems before the fact, and therefore, has recognized the need for a strong correlation program. Such a program has been maintained on a continuing basis by the Stability and Control Division over the past ten years. The overall objective of this program is to determine the accuracy with which the stability and control characteristics of submarines, surface ships, and other marine vehicles can be predicted by various alternative model and analytical techniques. In the more recent years, a large segment of the program has been devoted to establishing correlation between computer predictions based on techniques of the type described herein, and measurements taken on full-scale trials of submarines.

To provide meaningful correlation data for submarines, the need for designing and carrying out carefully conducted special types of full-scale trials cannot be overemphasized. In addition, to assure success, these special trials should be carried out as a complete package and not intermingled with operational maneuvers or other runs being performed for entirely different objectives. It is beyond the scope of this report to go into detail as to the manner in which these trials are conducted. Suffice it to say that the subject submarine must be highly instrumented with accurate sensors and recording equipment which may either supplement or be used in lieu of some of the ship's own instrumentation. Sufficient instrumentation coverage must be provided not only to measure the direct kinematic quantities involved, but also some of the more subtle quantities, which may affect later comparisons. Furthermore, in conducting such trials, strict attention must be paid to controlling initial or reference conditions including such precautionary measures as periodically taking "stop trims".

For a number of years, the Stability and Control Division has been conducting full-scale trials primarily to evaluate and establish numerical

measures of handling qualities of submarines. In general, these trials consist of definitive maneuvers of the types listed previously. The definitive maneuvers are programmed maneuvers carried out under controlled conditions in which measurements of all significant motion variables are taken with accurate instrumentation. Consequently, they are ideal for providing correlation data which are representative not only of definitive maneuvers, but most normal operational maneuvers as well. Included in the handling quality trials are some depthkeeping and coursekeeping runs at periscope depth under various sea states. The data from these tests can, at best, be used as a statistical tie-in with the computer predictions. Since the foregoing types of trials are usually carried out on the first of each new class of submarines, correlation data representative of normal maneuvers have been acquired on most of the different types of submarines that have been put into service from the AGSS 569 on.

Correlation data related to the various modes of motion associated with emergency recovery maneuvers have been considerably less abundant. This is because the emergency recovery maneuvers which were included in the handling-quality trials were conducted primarily to demonstrate the submarine's capability of recovering from sternplane jams of varying degrees. Consequently, accurate measurements were not taken of the various events and other quantities pertinent to the correlation problem. Accordingly, about 2½ years ago, it became necessary to design and initiate a series of special trials specifically directed toward providing the desired correlation data. These trials were conducted under the sponsorship of the Submarine Safety Program. They were of two types: one, to provide data on the behavior of representative submarines in sternplane jam recovery maneuvers; and the other, to provide data on behavior during buoyant ascents following blowing of the main ballast tanks. Obviously, in both types, the program was confined to maneuvers that could be carried out without endangering the submarine, as predetermined on the basis of computer predictions. Up to the present time, correlation trials of both the sternplane jam and emergency blow variety have been conducted on about six different submarines, although not necessarily the same ones in each case.

It will be several years before all of the data accumulated from the special trials of the various submarine types can be thoroughly analyzed and issued in a formal report containing a comprehensive treatment of the correlation problem for all of the modes of motion involved. In the interim, it is planned to issue separate reports for individual submarines which will compare measured trajectories with computed trajectories, based on the Standard Equations, for both normal and emergency maneuvers. These reports should serve to provide an indication of the current status of the prediction techniques. Some reports of this type have been issued for the cases of normal vertical-plane maneuvers, normal horizontal-plane maneuvers, and buoyant ascents associated with emergency blows. In addition, comparisons of measured and predicted trajectories were made, preparatory to conducting simulation studies to determine safe operating limits and emergency recovery capabilities, for about ten different submarine types. These comparisons included representative

normal maneuvers such as vertical overshoots, horizontal turns, and deceleration runs as well as some moderate sternplane jam emergency recovery maneuvers.

On the basis of the correlation studies and preliminary comparisons made to date, it appears that the Standard Equations together with coefficients of the type described herein will, for most part, yield accurate predictions of all pertinent trajectories associated with a variety of normal definitive or operational maneuvers in submerged ahead motion. In fact, the agreement in the majority of the cases investigated was well within the ability of the full-scale submarine to repeat the same maneuver. For emergency recovery maneuvers, generally good agreement has been found between measured and computed trajectories for moderate sternplane jam recoveries and buoyant ascents after ballast blows. Some unusual effects and discrepancies have been discovered while conducting emergency recovery maneuvers on a few of the submarines. Most of the discrepancies have been traceable to a variety of factors other than the hydrodynamic coefficients. However, certain unusual effects such as gross differences in behavior between the use of right and left rudder as a recovery measure, have not yet been adequately explained. In summary, within the range of the correlation studies made to date and in spite of the few discrepancies noted, there appears to be no valid reason to change the Standard Equations as they now exist.

The complete range of validity of the Standard Equations cannot be established solely on the basis of correlation studies of the types described herein. At best, these studies are limited to maneuvers that can be undertaken by the submarine with a reasonable margin of safety. For example, normal maneuvers for high-speed submarines are not carried out to their full potentiality. Also, the emergency recovery maneuvers are generally carried out at much less extreme conditions than those that could be encountered in real casualties. Consequently, it is necessary to use a somewhat different basis to assess the validity of the Standard Equations beyond the range covered by the correlation trials.

Basically, the range of validity of the Standard Equations could extend to all modes of motion encountered by a submarine during submerged flight at zero or any ahead speed. This would embrace various normal and emergency situations involving angles of attack or drift up to 90 degrees. To achieve this range, however, would require a complete characterization of the hydrodynamic forces and moments acting on a submarine model which covers a broad matrix of kinematic variables and combinations thereof, including various control-surface deflections. The characterizations currently being made, particularly those being used for the Submarine Safety Program, are fairly extensive and include angles of attack and drift up to 90 degrees. The results of such characterizations are, for most part, incorporated in the hydrodynamic coefficients in the Standard Equations. However, a simplified representation is still being used for the control coefficients associated with the deflection of sternplanes, bowplanes or sailplanes, and rudder. Although simplified, the representation for the control coefficients appears to give approximately

the same results as are obtained by a more sophisticated representation, even for fairly extreme maneuvers.

The simplified representation could become inadequate only if the effective angle of attack at a control surface becomes large, causing a condition of stall in which the lift no longer increases with the deflection angle. In normal submerged maneuvers, including steep dives and tight turns, the effective angles of attack at the control surfaces of a submarine tend to be substantially smaller than the nominal angles, and consequently stall will not be experienced. There are, however, some unusual conditions, such as may occur in certain flooding casualties, which could produce effective angles at the diving planes large enough to cause stall. If the condition is transient, and the large angle of attack is only of short duration, there may be little effect on the trajectories by using the more sophisticated representation. However, in studies of load supportability, large angles of attack can be obtained on the sternplanes, particularly where the load is concentrated at the stern. Since these studies involve a steady-state maneuver, the use of the simplified representation for sternplanes could be erroneous for the type of loading mentioned. Rather than encumber the Standard Equation to treat these few special cases, a separate sub-routine to represent diving plane stall has been developed and is available at NSRDC. The sub-routine represents the leveling off of lift, which is an approximate characteristic of control-surface stall, but allows the drag to increase with deflection angle.

In addition to the unusual, and as yet unexplained, effects which have been observed on full-scale submarines, there are several other factors which may be pertinent to the range of validity of the Standard Equations. Included among these are the effects of vortex shedding on rolling behavior, and effects due to the submarine moving close to the free surface while carrying large pitch, roll, or yaw angles of orientation. A research program to investigate these factors and which could possibly lead to future refinements and extensions of the Standard Equations, is being maintained at NSRDC.

CONCLUDING REMARKS

The Standard Equations presented in this report have been used successfully by NSRDC for the past 2½ years to conduct simulation studies of specific submarines engaged in submerged maneuvers ranging from normal maneuvers to extreme maneuvers such as those encountered in emergency recovery from sternplane jam and flooding casualties. During this period, several hundred of such studies involving about 25 different submarine designs have been carried out and the results of many of these studies are contained in the classified literature. A companion program of correlation studies is also being carried out to determine the extent to which computer predictions, involving the use of the Standard Equations, agree with measured trajectories obtained from full-scale trials of submarines undergoing various types of maneuvers. Those comparisons made to date have generally shown good agreement between

predicted and measured trajectories of representative maneuvers of both the normal and emergency types. It can be expected, therefore, that the NSRDC Equations used in conjunction with the specified hydrodynamic coefficients and sub-routines will provide simulations of trajectories that are well within the accuracy required for the purposes of those kinds of studies conducted up to the present time.

Although the NSRDC Standard Equations have reached a fairly advanced stage of development, they still must be considered only as representative of the state of the art, and subject to change if so dictated by future research. An active research program is being maintained at NSRDC to examine some of the remaining problem areas and to continue the development, if required. However, in interest of consistency, a policy has been established not to change the Standard Equations unless it can be demonstrated that significant improvements in predictions can be obtained. Such a change must be officially made by NSRDC, and all parties concerned will be properly notified.

In view of the foregoing, it is recommended that the NSRDC Standard Equations and associated methodology be adopted as a U.S. Navy-wide standard for simulation of specific submarine designs, and that they be given official status in dealing with contractors and other outside activities.

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The development of the NSRDC Standard Equations of Motion for Submarines and the associated experimental, analytical, and computer techniques required to implement them is the result of a strong team effort on the part of the members of the Stability and Control Division of the Hydromechanics Laboratory. Those members or former members of the Division deserving special mention are: Miss Elizabeth M. Dempsey who contributed to the refinement of the equations and development of techniques for estimating nonlinear coefficients; Mr. F.H. Imlay for identifying and systematizing cross-relationships for estimating couplings; Mr. Alex Goodman, co-inventor of the Planar-Motion-Mechanism System, who contributed strongly to the development of experimental techniques for determining the hydrodynamic coefficients; Mr. P.C. Clawson who devised and carried out full-scale trials to provide correlation data pertaining to normal and emergency recovery maneuvers which were needed to validate the primary mathematical models; and Mr. P.E. Markstrom and Mr. G.L. Santore who have been conducting correlation studies to validate the various mathematical models.

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Standard equations of motion are presented for use in submarine simulation studies being conducted for the U.S. Navy. The equations are general enough to simulate the trajectories and responses of submarines in six degrees of freedom resulting from various types of normal maneuvers as well as for extreme maneuvers such as those associated with emergency recoveries from sternplane jam and flooding casualties. Information is also presented pertaining to the hydrodynamic coefficients and other input data needed to perform simulation studies of specific submarine designs with the Standard Equations of Motion.

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